Spin polarization of phase delay time in a magnetic–electric barrier structure

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The spin-dependent phase delay time of two-dimensional electrons through a magnetic–electric barrier structure with/without an external electric field has been systematically investigated. The dependence of electron spin polarization on the applied bias, the incident wave vector, the incident electron energy and the height of the electric barrier has been addressed. It is found that the magnetic–electric barrier structure exhibits significant spin polarization under an external electric field, especially for electrons with small energies, where the spin polarization also displays a considerable wave vector-dependent feature.

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1 Introduction

Recently, the idea of electronic devices that exploit both the charge and spin of an electron for their operation has given rise to the new field of ‘spintronics’, literally spin electronics, in which the direction that an electron spin is pointing is just important as its charge. This nascent field has attracted considerable attention, fueled by the possibility of producing efficient photo-emitters with a high degree of polarization of the electron beam, creating spin-memory devices and spin transistors as well as exploiting the properties of spin coherence for quantum computation [1–12]. These structures are normally designed in the plane of a two-dimensional electron gas (2DEG) in the case where the magnetic flux lines of the microscopically inhomogeneous magnetic field thread this plane. This can be realized by fabricating magnetic dots or stripes and lithographic patterning of ferromagnetic materials and type-II superconductors on the semiconductor surface.

Although a few investigations have dealt with the electron spin problem in magnetically modulated quantum structures, almost all works have focused on the transmission probability and the conductance and their dependence on spin polarization. Related to the device performance, another important characteristic of the magnetic–electric structure is the intrinsic time scale, which is responsible for the speed of electronic devices. Although there are still debates on how to deal with the timing of an electron in tunneling processes [13], we will not attempt to discuss various concerns on this and different approaches. We will concern ourselves with the phase delay time and its dependence on spin polarization. The phase time is discussed in detail by Bohm [14] and Wigner [15], and making use of stationary-phase analysis provides the most direct approach for the study of the tunneling phenomenon [16–22].
In this work, we have studied theoretically the spin-dependent tunneling of two-dimensional electrons through a magnetic–electric barrier structure with or without an external electric field. We have derived analytical expressions for the phase time, which are given in simple and explicit form. Furthermore, the dependence of spin polarization on the applied bias, the electric barrier potential height, the incident wave vector and the incident electron energy is addressed systematically.

2 Theoretical calculations

We concentrate our attention on the simplest single electric–magnetic barrier structure, which permits the transmission probability calculation in closed form. We consider a 2DEG in the \((x, y)\) plane with a magnetic field \(B\) in the \(z\) direction. For generality, we adopt the same structure configuration as that in Ref. \[10\] including a rectangular electric barrier, as shown in Fig. 1b, where \(U(x) = U(\Theta(d/2 - |x|))\), and \(U(x)\) is considered as the total induced electrical potential in the 2DEG. We take the \(\delta\) function magnetic field of the form \(B = B_z(x)\) with \(B_z(x) = B[\delta(x + d/2) - \delta(x - d/2)]\), where \(B\) gives the strength of the magnetic field and \(d\) is the separation between the two \(\delta\) functions. We will concentrate our attention on this magnetic–electric barrier structure and will derive the phase delay time of electrons in this model potential configuration. The model potential configuration of Ref. \[9\], as shown in Fig. 1a, can thus be regarded as a special case here with \(U = 0\).

In the single effective mass approximation, the Hamiltonian describing such a system with an externally applied bias is given as

\[
H = \frac{1}{2m^*}[P + eA(x)]^2 + \frac{eg^*}{2m_0} \frac{\sigma \hbar}{2} B_z(x) - \frac{eV_x}{d},
\]

where \(m^*\) is the effective mass of the electron, \(m_0\) is the free-electron mass, \(V_x\) is the applied external bias, \(P\) is the momentum of the electron, \(g^*\) is the effective \(g\) factor of the electron in a real 2DEG realized using semiconductors, \(\sigma = \pm 1\) for up/down spin electrons and \(A(x)\) is the magnetic vector potential given in the Landau gauge by \(A(x) = B \Theta(d/2 - |x|)\). For simplicity, we introduce the dimensionless units of the electron cyclotron frequency \(\omega_c = eB_0/m^*\) and the magnetic length \(l_B = \sqrt{\hbar eB_0}\), where \(B_0\) is some typical magnetic field. We will express all the relevant quantities in dimensionless units by (i) the magnetic field \(B_z(x) \rightarrow B_0B_z(x)\), (ii) the vector potential \(A(x) \rightarrow B_0\omega_c A(x)\), (iii) the time \(t \rightarrow \omega_c^{-1}t\), (iv) the coordinate \(r \rightarrow l_B r\), and (v) the energy \(E \rightarrow \hbar \omega_c E\). For GaAs and an estimated \(B_0 = 0.1 \, T\) and \(m^* = 0.067\), we have \(l_B = 813 \, \text{Å}, \, \omega_c = 0.387 \times 10^{-17} \, \text{s}, \, E_0 = \hbar \omega_c = 0.17 \, \text{meV}, \, g^* = 6.6\) [23].

Because the system is translation invariant along the \(y\) direction, the solution of the stationary Schrödinger equation \(H \Psi(x, y) = E \Psi(x, y)\) can be written as a product \(\Psi(x, y) = e^{ik_y y} \varphi(x)\), where the wave
function \( \psi(x) \) satisfies the one-dimensional Schrödinger equation:

\[
\left[ \frac{d^2}{dx^2} - k_x^2 + 2E \right] \psi(x) = 0 \quad \text{for } x > |d/2|,
\]

\[
\left[ \frac{d^2}{dx^2} - (k_x + A(x))^2 - \frac{m^* g^* \sigma B\,(x)}{2m_0} + \frac{2(E-U)(x) + 2eV_{\alpha}x}{d} \right] \psi(x) = 0 \quad \text{for } x < |d/2|.
\]

It is important to introduce the effective potential \( U_{\text{eff}}(x, k_y, \sigma) = [A(x) + q]/2 + m^* g^* \sigma B\,(x)/4m_0 + eV_{\alpha}x/2 \) of the corresponding structure in the barrier region, which depends not only on the magnetic configuration, the wave vector \( k_x \), but also on the interaction between the electron spin and the non-homogeneous magnetic field. It is noted that the last in the effective potential is zero everywhere except at \( x = \pm d/2 \).

2.1 Without bias

First we consider the case without bias. Following the standard procedure outlined in quantum mechanics texts we can easily obtain the phase delay \( \Phi(E, k_y, \sigma) \) in a magnetic–electric barrier structure:

\[
\Phi(E, k_y, \sigma) = \arctan \left( \frac{k_1 + k_2^2 + (m^* \sigma B/2m_0)^2}{2k_2} \tan k_x d \right) \quad \text{for } 2(E-U) > (k_x + B)^2,
\]

\[
\Phi(E, k_y, \sigma) = \arctan \left( \frac{k_1 - k_2^2 + (m^* \sigma B/2m_0)^2}{2k_2} \tanh k_x d \right) \quad \text{for } 2(E-U) < (k_x + B)^2.
\]

with \( k_1 = \sqrt{2E - k_y^2} \) and \( k_2 = \sqrt{2(E-U) - (k_x + B)^2} \).

The phase delay time is defined as [14, 15]

\[
\tau = \partial \Phi/\partial E.
\]

By inserting Eq. (3) into Eq. (4) we get

\[
\tau = \frac{k_1k_2}{k_1k_2} \left[ 4k_1^2k_2^2 \cos h^2(k_x d) + \left[ (k_1^2 + (m^* \sigma B/2m_0)^2 - k_2^2) \sinh(k_x d) \right]^2 \right]
\]

for \( 2(E-U) < (k_x + B)^2 \),

\[
\tau = \frac{k_1k_2}{k_1k_2} \left[ 4k_1^2k_2^2 \cos^2(k_x d) + \left[ (k_1^2 + (m^* \sigma B/2m_0)^2 + k_2^2) \sin (k_x d) \right]^2 \right]
\]

for \( 2(E-U) > (k_x + B)^2 \).

2.2 Under an applied bias

Under an applied bias the solutions of the Schrödinger equation (Eq. (2)) are the well-known linearly independent Airy functions [24] \( A_i(x) \) and \( B_i(x) \). By using a similar procedure to that above we obtain the phase delay of spin electrons in a magnetic–electric barrier structure under an external applied bias:

\[
\Phi = \arctan \left[ \frac{\kappa/\lambda - (m^* \sigma B/2m_0)(\gamma + \beta) + \lambda\alpha(k_1k_2 + (m^* \sigma B/2m_0)^2)}{k_1\beta - k_2\gamma + \lambda\alpha(m^* \sigma B/2m_0)(k_x - k_y)} \right]
\]
with
\[ \begin{align*}
\alpha &= A_1[\rho(0)]B_1[\rho(d)] - B_1[\rho(0)]A_1[\rho(d)], \\
\beta &= A_2[\rho(0)]B_2[\rho(d)] - B_2[\rho(0)]A_2[\rho(d)], \\
\gamma &= A_3[\rho(0)]B_3[\rho(d)] - B_3[\rho(0)]A_3[\rho(d)], \\
\kappa &= A_4[\rho(0)]B_4[\rho(d)] - B_4[\rho(0)]A_4[\rho(d)],
\end{align*} \]  
(7)

where
\[ \rho(x) = \frac{1}{\lambda} \left[ x + \frac{d}{(eV_x)} \right] \left[ E - U - (k_x + B)^2 / 2 \right] \],
\[ \lambda = \left( -\frac{d}{2eV_x} \right)^{1/3}, \quad k_x = \sqrt{2E - k_y^2}, \quad k_y = \sqrt{2(E + eV_x) - k_x^2}. \]

The phase delay time is the same as defined in Eq. (4). With the phase delay time the electron spin polarization effect can be evaluated by
\[ P = \frac{\tau_+ - \tau_-}{\tau_+ + \tau_-}, \]
(8)

where \( \tau_+ \) and \( \tau_- \) are the phase delay times of electrons with spin up and spin down, respectively.

3 Results and discussion

Figure 2 plots the phase delay time as a function of incident electron energy with electron spin up/down and without considering the electron spin respectively for different electron momentum \( K_y \) and different values of the electric potential barrier height \( U \). We assume \( d = 0.5 \) and \( B = 3.5 \) in our calculations. It is
seen clearly that the phase time is altered significantly for all values of energy with the introduction of the spin–magnetic field interaction and the rectangular electric potential. For the electric–magnetic barrier structure without bias, the phase delay time is always enhanced by the spin–magnetic interaction. The phase delay time is reduced for positive $U$ but is enhanced for negative $U$. Furthermore, both a magnetic barrier and an electric barrier exhibit significant wave vector filtering properties. However, the phase delay times for both spin-up electrons and spin-down electrons are always the same. This indicates clearly that both a magnetic barrier and an electric barrier structure without bias are unable to distinguish the two possible spin states of the electrons. This finding is definite and of course results from the phase delay time of Eq. (5), where the electron spin $\sigma$ appears in the equation as $\sigma^2$.

Figure 3 shows the spin polarization versus incident electron energy for different electron momentum $k_y$ and different values of the electric potential barrier height $U$ under a certain applied bias of 2.0. Under an applied bias the phase delay time is significantly altered; the electrons show considerable spin polarization, especially for electrons with small energies. It is also noted that the phase delay time for an electron with spin down is always longer than that of an electron with spin up, and the spin polarization can be tuned by the electric barrier. For large electronic energy, the spin polarization is weakened, and finally approaches zero. It is also noted that the electron shows stronger wave vector-dependent spin polarization for small electronic energies.

Figure 4 shows the spin polarization versus the applied bias for different electron momentum $k_y$ and different values of the electric potential barrier height $U$ at a certain incident energy of 2.0. We see that the spin polarization magnitude changes greatly under the influence of the applied bias; it is greatly strengthened with increasing applied bias. It is also seen clearly that for different wave vector $k_y$, the spin polarization is very different, and as the magnitude of the wave vector $k_y$ increases the spin polarization is weakened. Moreover, upon further increasing the applied bias, the spin polarization finally saturates at different values for different wave vector $k_y$. 
4 Conclusions

We have derived analytical expressions for the phase time, which are given in simple and explicit form. The dependence of spin polarization on the applied bias, the electric barrier potential height, the incident wave vector and the incident electron energy is addressed systematically. It is found that the spin electron in a magnetic barrier and an electric barrier structure exhibits significant spin polarization and spin filtering under an applied bias, especially for electrons with small energies. The spin polarization depends strongly on the applied bias, the electric barrier potential height, the electron kinetic momentum and the incident electron energy. However, both a magnetic barrier and an electric barrier without bias do not possess any spin polarization or spin filtering.

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References